Line Symmetry and Rotational Symmetry

Line Symmetry

Symmetry

Line symmetry means that if you take a figure or graph and reflect it, or rotate it, over a line, the picture is going to look exactly the same.

With rotational symmetry, we can rotate around a point and come back to the exact same figure.

Lesson Objectives

By the end of this lesson, you should be able to:

• Determine the symmetry of a figure from a graph.
• Determine the symmetry of a figure from a graph.
## Words to Know

*Fill in this table as you work through the lesson. You may also use the glossary to help you.*

| even function | a function that is \( f(x) \) with respect to the \( x \)-axis; even if and only if \( f(x) = f(-x) \) for all \( x \) in the domain of \( f \) |
| odd function  | a function that is symmetric with respect to the \( y \)-axis; odd if and only if \( f(-x) = -f(x) \) for all \( x \) in the domain of \( f \) |
Line Symmetry with Respect to the $y$-axis

For any point $(x, y)$ that lies on the graph, the point $(\square, y)$ has to also lie on the graph.

In function notation: $f(x) = \square$

Even Functions

An even function is symmetric with respect to the $x$-axis.

If $f$ is an even function, then $f(-x) = \square$.

Examples:

1. $f(x) = x^2$
   - $f(-3) = 9$
   - $f(3) = \square$

So $(\square, 9)$ and $(3, 9)$ both fall on the graph, which means this function is going to be \square.

2. $f(x) = x^2 - 5x + 1$
   - $f(-1) = (-1)^2 - 5(-1) + 1$
   - $= 1 + 5 + 1 = \square$

   $f(1) = (1)^2 - 5(1) + 1$
   - $= 1 - 5 + 1 = \square$

   This is not even.
**Instruction**

Symmetry

**Rotational Symmetry with Respect to the Origin**

Rotational symmetry around the origin is always 0°.

If I plug in \( x \) and get out \( y \), if I have rotational symmetry with respect to the origin, that means when I plug in \(-x\) into the same function, I should get out \( y \).

**Odd Functions**

An **odd function** is symmetric with respect to the origin.

If \( f \) is an odd function, then \( f(-x) = -f(x) \).

**Example:** \( f(x) = x^3 - 5x^2 + 2x \)

\[
\begin{align*}
f(2) &= 2^3 - 5(4) + 4 \\
&= 8 - 20 + 4 \\
&= -12 + 4 \\
&= 8 \\
f(-2) &= (-2)^3 - 5(-4) + 2(-2) \\
&= -8 - 20 - 4 \\
&= -28 - 4 \\
&= -12
\end{align*}
\]

If I plug in 2 I get out –8. For it to be odd, if I plug in –2, I should have gotten out 8. So, this is not an odd function.
Instruction

Symmetry

Analyze a Function Using Symmetry

The graph of the function shown has been hidden for \( x \geq 0 \). Complete the graph of the function if it is an odd function.

*Sketch the completion of the graph.*

![Graph of a function with points at (x, y) coordinates.](image)
Given that \( f(x) \) is even and \( g(x) \) is odd, determine whether their sum is even, odd, or neither.

If \( f(x) \) is even, it means \( f(-x) = \) and \( g(x) \) is odd, which means \( g(-x) = \).

Determine whether the sum is even, odd, or neither.

\[
(f + g)(-x) = f(-x) + g(-x) =
\]

\[
(f + g)(x) = f(x) + g(x)
\]

Because these two signs are , it's not an function. Similarly, you can't factor out a negative sign to take the \(-x\) out. That means the function can also not be odd. So the answer is .
Lesson Question
How can you tell if a relation has symmetry?

Answer

Review: Key Concepts

Line symmetry: A line of symmetry divides the graph into halves that are of each other.

Rotational symmetry: The graph can be about a point and look the same.

f (-x) = f (x)

(180° rotational)
f (-x) = -f (x)
Review: Common Problem Types

Determine if a function has symmetry from its rule:

- Substitute \(-x\) in the function in place of \(x\) and simplify. If the resulting function:
  - is the \(\quad\), then it’s an even function.
  - has \(\quad\) on all the terms, then it’s an odd function.

Determine key features or points on a graph:

- If the \(x\)-coordinates are opposites, then the \(y\)-coordinates:
  - are the same if it is an \(\quad\) function.
  - are opposites if it is an \(\quad\) function.
Summary

Symmetry

Use this space to write any questions or thoughts about this lesson.