Lesson Goals

Apply the formulas for volume of a sphere and a cube.

Identify perfect cubes.

Understand

Determine side length and measures.

Words to Know

Write the letter of the definition next to the matching word as you work through the lesson. You may use the glossary to help you.

_____ approximate A. set of all points in space a given distance from a fixed point

_____ perfect cube B. a number that is the result of cubing a natural number

_____ radius C. near or very close to the actual

_____ sphere D. a segment that extends from the center of a circle to any point on the circle

_____ volume E. the measure of the amount of space occupied by a three-dimensional solid object
Volume of a Sphere

The formula for the volume, \( V \), of a sphere is given by \( V = \frac{4}{3} \pi r^3 \).

- What is the volume of a sphere with a diameter of 6 mm?

\[
V = \frac{4}{3} \pi r^3
\]

\[
V = \frac{4}{3} \pi (3)^3
\]

\[
V = \frac{4}{3} \pi (27)
\]

\[
V = (4)(\underline{27})\pi
\]

\[
V = \underline{36}\pi \text{ mm}^3
\]
Spherical and Cubic Volume Applications

**Calculate the Volume**

What is the volume of this cube with a side length of 5 centimeters?

- Cube \( V = s^3 = s \cdot s \cdot s \)

\[
V = \boxed{s^3} = 5 \cdot 5 \cdot 5 = \boxed{cm^3}
\]

**Find the Side Length**

**EXAMPLE**

What is the side length of a cube with a volume of 8 cm\(^3\)?

- Cube \( V = s^3 \)

\[
8 = \boxed{s^3} \quad \Rightarrow \quad \boxed{\sqrt[3]{s^3}} = cm = s
\]
The Root Versus Dividing

MISCONCEPTIONS

What is the side length of a cube with a volume of 27 mm$^3$?

Taking the root:

- $\sqrt[3]{27} = 3$ because $3^3 = 3 \cdot 3 \cdot 3 = 27$

- $V = 3^3 = 3 \cdot 3 \cdot 3 = \boxed{27}$ mm$^3$

Correct method.

Dividing:

- $\frac{27}{3} = 9$ because $27 \div 3 = 9$

- $V = 9^3 = 9 \cdot 9 \cdot 9 = \boxed{729}$ mm$^3$

So this method of dividing is incorrect.
### Perfect Cubes versus Not-Perfect Cubes

#### Perfect cubes
- 1 unit\(^3\)
  \[1 \cdot 1 \cdot 1 = 1\]
- 64 units\(^3\)
  \[4 \cdot 4 \cdot 4 = 64\]
- 512 units\(^3\)
  \[8 \cdot 8 \cdot 8 = 512\]
- 729 units\(^3\)
  \[9 \cdot 9 \cdot 9 = 729\]

#### Not-perfect cubes
- 4 units\(^3\)
  \[1^3 < 4 < 2^3\]
  \[1 < 4 < 8\]
- 12 units\(^3\)
  \[2^3 < 12 < 3^3\]
  \[8 < 12 < 27\]
- 25 units\(^3\)
  \[< 25 < 3^3\]
  \[8 < 25 < 27\]
- 600 units\(^3\)
  \[8^3 < 600 < 9^3\]
  \[512 < 600 < 729\]

Why is it not a perfect cube? It falls between two.
Find the Radius of a Sphere

**EXAMPLE**

The volume of a sphere is $36\pi$ cubic centimeters. What is the radius?

- Sphere $V = \frac{4}{3}\pi r^3$

\[
V = \frac{4}{3}\pi r^3
\]

\[
\left(\frac{3}{4}\right)(36)\pi = \left(\frac{3}{4}\right)\left(\frac{4}{3}\right)\pi r^3
\]

\[
\frac{27}{\pi} = \frac{\pi r^3}{\pi}
\]

\[
27 = r^3
\]

\[
\sqrt[3]{27} = \sqrt[3]{r}
\]

\[
3 = r
\]

\[
r = 3 \text{ cm}
\]
Volume of a Basketball

REAL-WORLD CONNECTION

A basketball has a radius of 5 inches. **Approximately** how much air does it take to fill the basketball?

- Sphere \( V = \frac{4}{3} \pi r^3 \)

\[
V = \frac{4}{3} \pi r^3 \\
V = \frac{4}{3} \pi (5)^3 \\
V = \left(\frac{4}{3}\right) \pi \text{ in.}^3 \\
V \approx \text{ in.}^3
\]
Find the Diameter of a Real-World Sphere

Marcel received a globe as a gift. On the box, it states that it has a volume of $972\pi$ cubic inches. He wants to find the diameter measure to make sure that it will fit in his shelf space before he opens the box. What is the diameter of Marcel's globe?

$$V = \frac{4}{3} \pi r^3$$

$$\left(\frac{3}{4}\right)(972)\pi = \left(\frac{4}{3}\right)\pi r^3$$

$$\pi \frac{r^3}{\pi} = \frac{r^3}{r^3}$$

$$729 = r^3$$

$$\sqrt[3]{729} = \sqrt[3]{r^3}$$

$$r = 9 \text{ because } 9^3 = 729$$

$$d = 9 \cdot \frac{r}{2} = 18 \text{ in.}$$

The diameter of Marcel's globe is $18 \text{ inches}$. 
Summary
Spherical and Cubic Volume Applications

Lesson Question
How can you apply the formulas for volume of a cube and a sphere to solve problems?

Answer

Use this space to write any questions or thoughts about this lesson.