

Warm-Up

Recognizing Patterns



Lesson Question



Lesson Goals

Analyze patterns to if a sequence is arithmetic, geometric, or neither.

Calculate a of a sequence.

Create a recursive for a sequence.



Words to Know

Fill in this table as you work through the lesson. You may also use the glossary to help you.

arithmetic sequence	a sequence in which the <input type="text"/> between any two consecutive terms is constant
geometric sequence	a sequence in which the <input type="text"/> between any two consecutive terms is <input type="text"/>
recursive formula	the formula used to <input type="text"/> the terms of a recursive sequence



Words to Know

recursive sequence	a sequence in which any term is determined by a function of <input type="text"/> terms
sequence	a set of <input type="text"/> numbers
term	the sequence value at a specific <input type="text"/>



Evaluating Functions

- Table

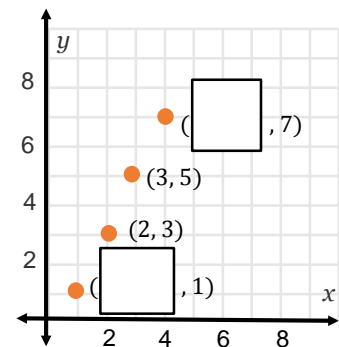
x	y
1	1
2	3
3	5
4	7

- Function rule

$$f(x) = 2x - 1$$

$$\begin{aligned}
 f(1) &= 2(\text{input}) - 1 \\
 &= 2 - 1 \\
 &= \text{output}
 \end{aligned}$$

- Graph



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Sequences

- A **sequence** is a set of numbers.
- sequence:
 - 4.5, 5.6, 6.7, 7.8
 - add 1.1
- sequence:
 - 4.5, 5.6, 6.7, 7.8, ... ,
 - ...

Sequence Notation

- A , a_n , is the sequence value at a specific position, n .

<input type="text" value="4.5"/>	<input type="text" value="5.6"/>	<input type="text" value="6.7"/>	<input type="text" value="7.8"/>
• <input type="text"/> term	• 2nd term	• 3rd <input type="text"/>	• 4th term
• $n = 1$	• $n =$ <input type="text"/>	• $n = 3$	• $n = 4$
• $a_1 = 4.5$	• $a_2 = 5.6$	• $a_3 = 6.7$	• $a_4 =$ <input type="text"/>

Instruction

Recognizing Patterns

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Graphing a Sequence

A sequence can be graphed as a

function where the:

- domain is a subset of the numbers.

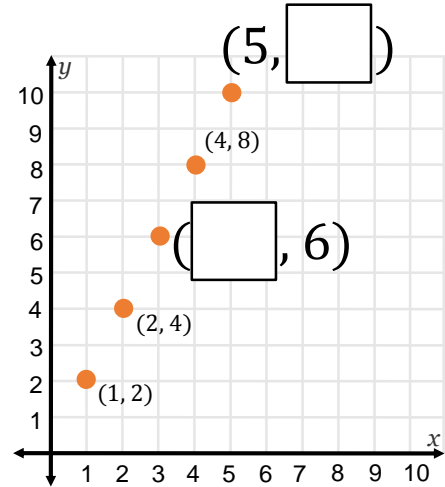
{1, 2, 3, 4, 5}

- range is the of the sequence.

{2, 4, 6, 8, 10}

The sequence is 2, 4, 6, 8, 10.

Each coordinate can be thought of as (n, a_n) .



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Relating Sequences and Functions

n

$$a_n = f(n)$$

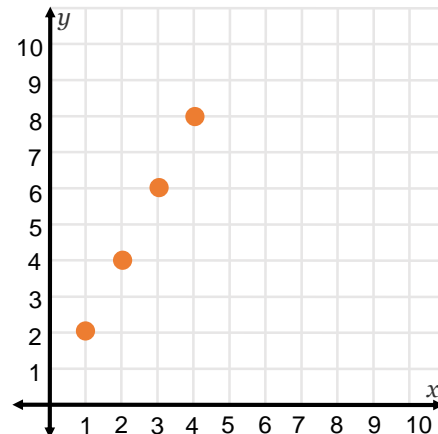
x	y
1	2
2	4
3	6
4	8
5	10

$$f(\text{ })$$

$$f(2)$$

$$f(3)$$

$$f(4)$$



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Identifying Patterns

Describe the relationship between the numbers in the sequences. Then, extend the patterns.

<u>pattern</u>	<u>rule</u>	<u>next term</u>
• 2, -4, 8, -16, ...	multiply by <input type="text"/>	32
• 1, 2, 2, 4, 8, 32, ...	multiply two previous terms	<input type="text"/>
• 2, 3, 5, 7, 11, 13, ...	<input type="text"/> numbers	17
• $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \dots$	$\frac{1}{\underline{\quad}}$	<input type="text"/>

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Recursive Sequences

- In a **recursive sequence**, any term is determined by a function of

terms.

- 1, 1, 2, 3, 5, 8, 13, 21, ...

Fibonacci sequence

Recursive formula: $a_3 = a_1 + \text{$

Function notation: $f(3) = f(1) + f(2)$

$$= 1 + 1$$

$$= \text{$$

$$a_4 = a_2 + a_3$$

$$f(4) = f(2) + f(3)$$

$$= 1 + 2$$

$$= 3$$

- A recursive sequence can be defined by a **recursive** .

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A recursive formula allows us to find the n th term, any term a_n , as long as you know the two terms before it.

Recursive formula: $a_n = a_{n-2} + a_{n-1}, a_1 = 1 \quad a_2 = 2$

Function notation: $f(n) = f(n-2) + f(n-1)$ for $n \geq$
 $f(1) = 1, f(2) = 1$

We could also state: $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$

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Arithmetic and Geometric Sequences

ARITHMETIC SEQUENCES

In an **arithmetic sequence**, there is a common

between any two consecutive terms.

6.9, 1.9, -3.1, -8.1, ...

$$-3.1 - 1.9 = \text{$$

$$-1.9 - 6.9 = \text{$$

To get to the next term, I have to 5 for each one of these terms.

GEOMETRIC SEQUENCES

In a **geometric sequence**, there is a common between any two consecutive terms.

• 2, -4, 8, -16 ...

$$\frac{-4}{2} = \text{$$

$$\frac{8}{-4} = \text{$$

• 100, 20, 4, $\frac{4}{5}$

$$\frac{20}{100} = \text{$$

$$\frac{4}{5} \div 4 = \frac{4}{5} \cdot \frac{1}{4} = \text{$$

Not every sequence is arithmetic or geometric. Some sequences don't have a pattern, like the digits of π . In order for the sequence to be arithmetic or geometric, remember you're looking for either a common difference, or a common ratio.

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Recursive Formulas for Arithmetic and Geometric Sequences

Arithmetic Sequences

$$f(n+1) = f(n) + \boxed{}, n \geq 1$$

$$f(n) = c$$

- $\frac{4}{5}, 1, 1\frac{1}{5}, 1\frac{2}{5}$

Find the common difference:

$$1 - \frac{4}{5} = \frac{5}{5} - \frac{4}{5} = \frac{1}{5}$$

$$d = \boxed{}$$

Write the general form:

$$f(n+1) = \boxed{} + \frac{1}{5}, n \geq 1$$

$$f(1) = \boxed{}$$

Geometric Sequences

$$f(n+1) = \boxed{} f(n), n \geq 1$$

$$f(n) = c$$

- 2, -4, 8, -16 ...

Find the common ratio:

$$r = \boxed{}$$

Write the general form:

$$f(n+1) = \boxed{} \cdot (-2)$$

$$f(n+1) = (-2) f(n), n \geq \boxed{}$$

$$f(n) = 2$$

Summary

Recognizing Patterns

**Lesson
Question**

How are functions used to describe patterns?

**Answer**

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2**Review: Key Concepts**

- A sequence can be as a discrete function where the domain is a subset of the natural numbers.
- $f(n) = a_n$
- A term of a recursive sequence is determined by a of previous terms.
 - sequences: $f(n + 1) = f(n) + d$
 - Geometric sequences: $f(n + 1) = rf(n)$

Use this space to write any questions or thoughts about this lesson.