Warm-Up | Quadratic Functions: Vertex Form

Lesson Question

What does the vertex form of a quadratic function reveal about its relationship to $y = x^2$?

Lesson Goals

Graph quadratic functions in vertex form.

Identify the vertex and axis of symmetry.

Analyze the graph for key features.

Graph vertical and horizontal translations.

Words to Know

Fill in this table as you work through the lesson. You may also use the glossary to help you.

axis of symmetry	in a parabola, the line that passes through the vertex and is the line of reflection
parabola	the U-shaped graph of a quadratic function; the set of points that is equidistant from a given point and a given line
translation	a transformation that shifts the graph of a parent function vertically or horizontally without changing the shape of the graph
vertex	in a parabola, the point at which the function goes from increasing to decreasing or vice versa

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Standard Form of a Quadratic Function

The standard form of a quadratic function is $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

For a quadratic function in standard form, the **vertex** is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

- Determine the vertex of the function

  $f(x) = -2x^2 + 8x - 1.$

  $a =$ −2 $\quad b =$ 8

  $-\frac{b}{2a} = \frac{-8}{2(-2)} =$ 2 x-value

  $f(2) = -2(2)^2 + 8(2) - 1$

  $= -8 + 16 - 1 =$ 7 y-value

vertex : (2, 7)

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Vertex Form of a Quadratic Function

The vertex form of a quadratic function is $f(x) = a(x - h)^2 + k$.

- The **parabola** opens up if $a > 0$ and opens down if $a < 0$.
- The vertex is located at (h, k).
- The **axis of symmetry** is the line $x = h$.

Complete the statements for the function $g(x) = -3.5(x + 5)^2 - 4$.

The parabola opens down.

The vertex is located at (−5, −4).

The axis of symmetry is the line $x =$ −5.

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Comparing Different Forms of a Quadratic Function

Vertex Form

$f(x) = (x - 2)^2 - 9$

x	y
−1	0
0	−5
1	−8
2	−9
3	−8

$f(1) - (1 - 2)^2 - 9$
$= (-1)^2 - 9$
$=$ −8

$(x - 2)^2 - 9$
$(x - 2)(x - 2) - 9$
$x^2 - 2x - 2x + 4 - 9$
$x^2 - 4x - 5$

Standard **Form**

$f(x) = x^2 - 4x - 5$

x	y
−1	0
0	−5
1	−8
2	−9
3	−8

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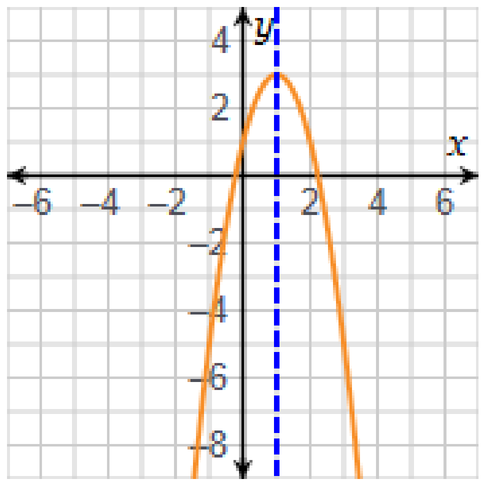
How to Graph a Quadratic Function Given in Vertex Form

Graph the function $g(x) = -2(x-1)^2 + 3$.



1. Identify a, h, and k.
   - $a =$ −2 $, h = 1, k =$ 3
2. Plot the vertex.
   - (1 $, 3)$
3. Draw the axis of symmetry.
   - $x = 1$
4. Evaluate the function at two other x-values, and plot the points. Use symmetry to plot corresponding points.
   - $g(0) =$ 1 $, g(-1) =$ −5
5. Connect the points with a smooth curve to form the parabola.

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Analyzing a Graph for Key Features

The function $h(x) = -\frac{1}{6}(x+3)^2 + 6$ is graphed.

Identify these key features:

$h = -3$ $k = 6$

- Vertex

  $(-3,$ 6 $)$

- Axis of symmetry

  $x =$ −3

- Domain (x)

  $\{x | x$ is a real number$\}$

- Range (y)

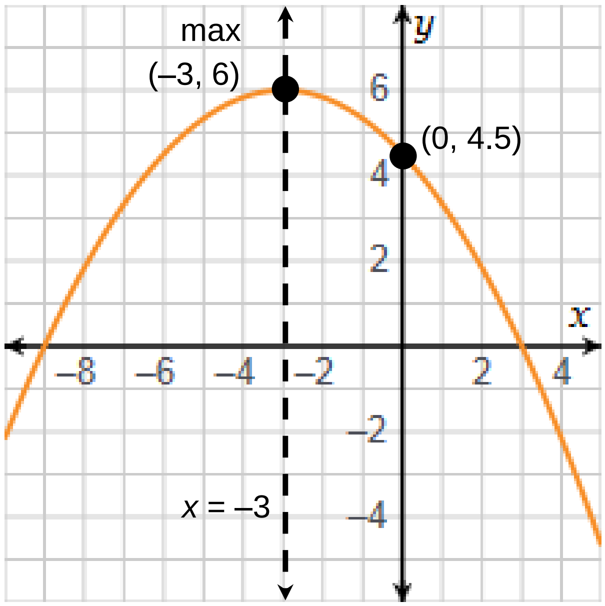
  $\{y | y \leq$ 6

- y-intercept

  $h(0) = \frac{27}{6} =$ 4.5

- Increasing and decreasing intervals

  $(-\infty,$ −3 $), (-3,$ ∞ $)$



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Reflection across the x-Axis

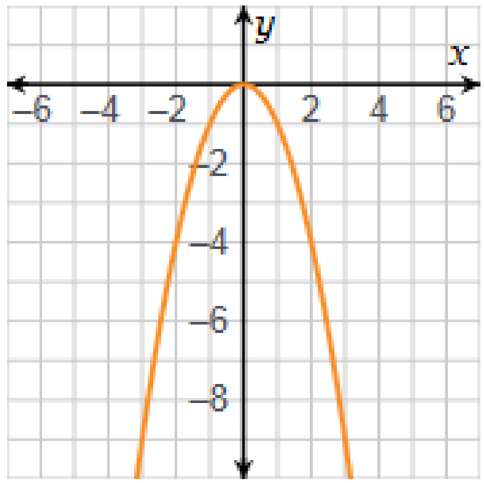
Graph the function $h(x) = -x^2$.

$a =$ -1 $\qquad h = 0, k = 0 \rightarrow (0, 0)$

Opens Down

$h(-1) = -(-1)^2 = -1 \rightarrow (-1,$ -1 $)$

$h(-3) = -(-3)^2 = -9 \rightarrow ($ -3 $, -9)$



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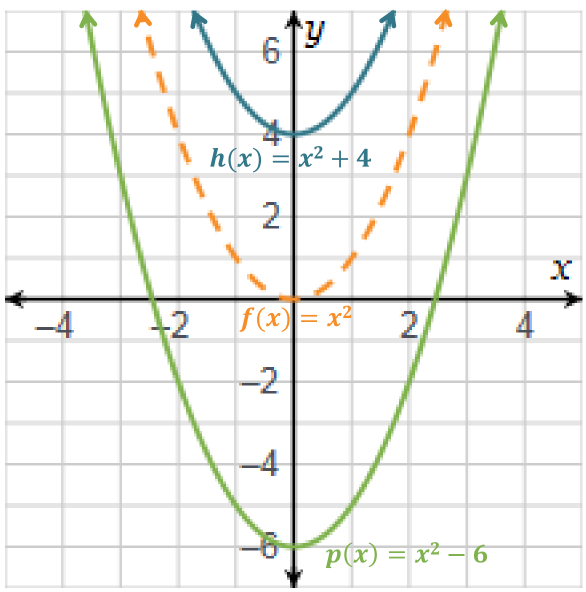
Vertical Translations

The graph of $g(x) = x^2 + k$ is the graph of $f(x) = x^2$ translated vertically.

- If k is positive, the graph of $f(x) = x^2$ is translated up k units.
- If k is negative, the graph of $f(x) = x^2$ is translated down k units.



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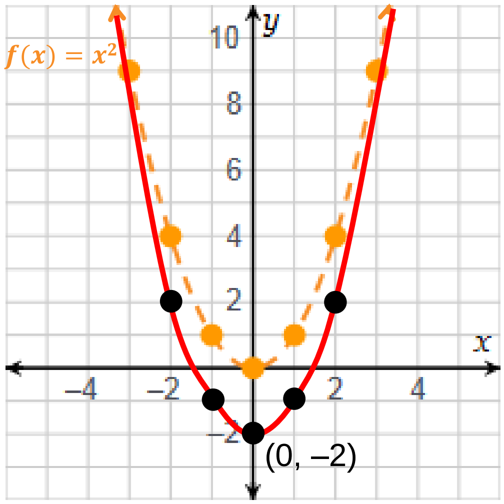
Using a Vertical Translation to Graph a Function

Identify the **translation** and graph the function $g(x) = x^2 - 2$.

$k =$ −2

The translation is down 2 units.

Graph $g(x)$.



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Horizontal Translations

The graph of $g(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.

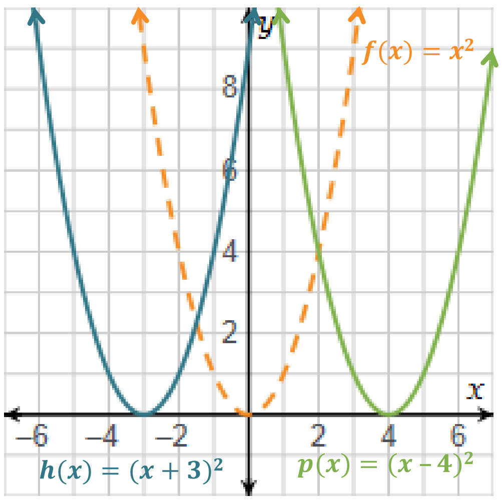
- If h is positive, the graph of $f(x) = x^2$ is translated right h units.
- If h is negative, the graph of $f(x) = x^2$ is translated left h units.

$h = 4$: $(x - 4)^2$

Shift to the right 4 units.

$h = -3$: $(x - (-3))^2 = (x + 3)^2$

Shift to the left 3 units.



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Graphing Vertical and Horizontal Translations

Graph $p(x) = (x - 2)^2 - 4$.

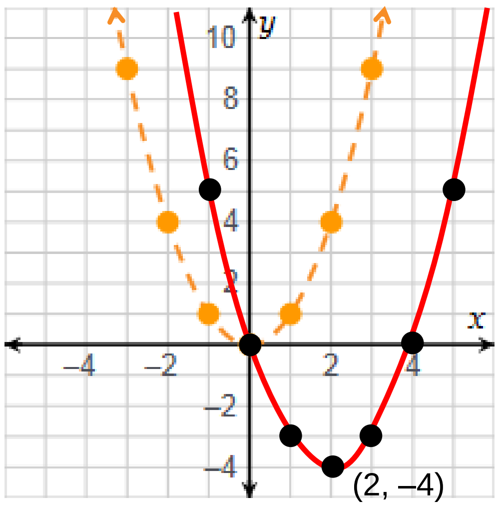
$h = 2$

The graph is going to shift right 2 units.

$k = -4$

The graph is going to move down 4 units.

Graph p(x).



Summary | Quadratic Functions: Vertex Form

Lesson Question

What does the vertex form of a quadratic function reveal about its relationship to $y = x^2$?

Answer

(Sample answer) The vertex form of a quadratic function gives the vertex and shows translations, stretches, and shrinks from the parent function.

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Review: Key Concepts

The quadratic function $g(x) = a(x - h)^2 + k$ is in vertex form, and is a transformation of $f(x) = x^2$.

- The sign of a determines if the parabola opens up or down.
- The absolute value of a determines the shape of the graph.
- (h, k) is the vertex of the parabola.
  - The value of h determines if the parabola shifts left or right.
  - The value of k determines if the parabola shifts up or down.

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Use this space to write any questions or thoughts about this lesson.