Warm-Up | Rotations

Lesson Question

How are rotations defined mathematically?

Lesson Goals

Develop the formal [definition] of a rotation.

Write a [rule] that describes a given rotation.

Determine the image or pre-image of a figure after a given [rotation] .

Words to Know

Fill in this table as you work through the lesson. You may also use the glossary to help you.

angle of rotation	the [measure] of the angle between a ray drawn from the center of rotation to a point on the pre-image and a ray drawn from the [center] of rotation to the corresponding point on the image
center of rotation	in a rotation, the fixed [point] about which the figure is turned
image	the result of a transformation of a geometric [shape]
pre-image	a geometric shape [before] a transformation occurs

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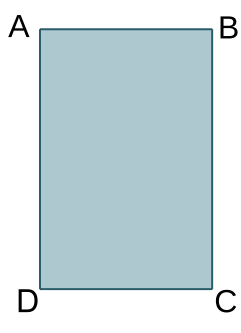
Words to Know

rigid transformation	a transformation that preserves the size, length, shape, lines, and angle measures of a figure
rotation	in a plane, a transformation in which each point on a figure is turned through a given angle and direction around a given point called the center of rotation such that the distance between the point and the center of rotation remains fixed

Transformations of Figures

Rectangle ABCD can be translated left or right and up or down. What if we wanted to move the rectangle so that it rests on one of the longer sides?



To turn the rectangle we would use a rotation .

Rotations allow us to turn , or flip, figures.

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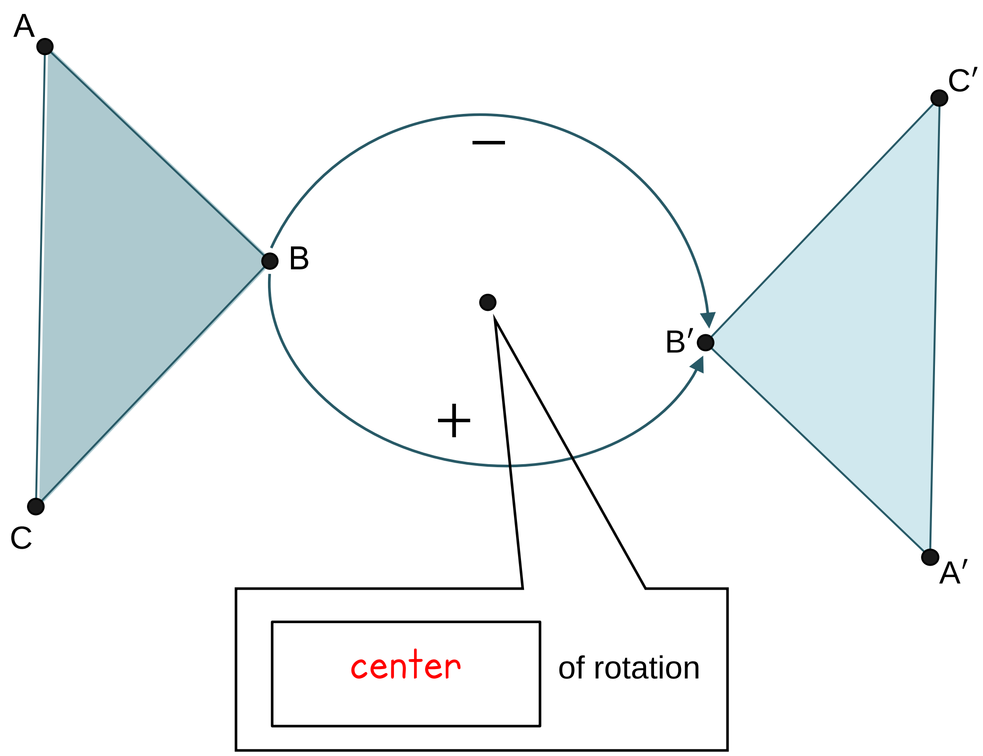
Rotations: Definition

A rotation is a **rigid transformation**, sometimes called an isometric transformation, that moves every point of the **pre-image** through an angle **of rotation** about the **center of rotation** to create an **image**.

- The points are rotated in a given direction, either clockwise or counterclockwise, around the center of rotation.
- When angles of rotations are counterclockwise from the center of rotation, the angle is positive.
- If the figure is rotated clockwise, the angle of rotation is negative.



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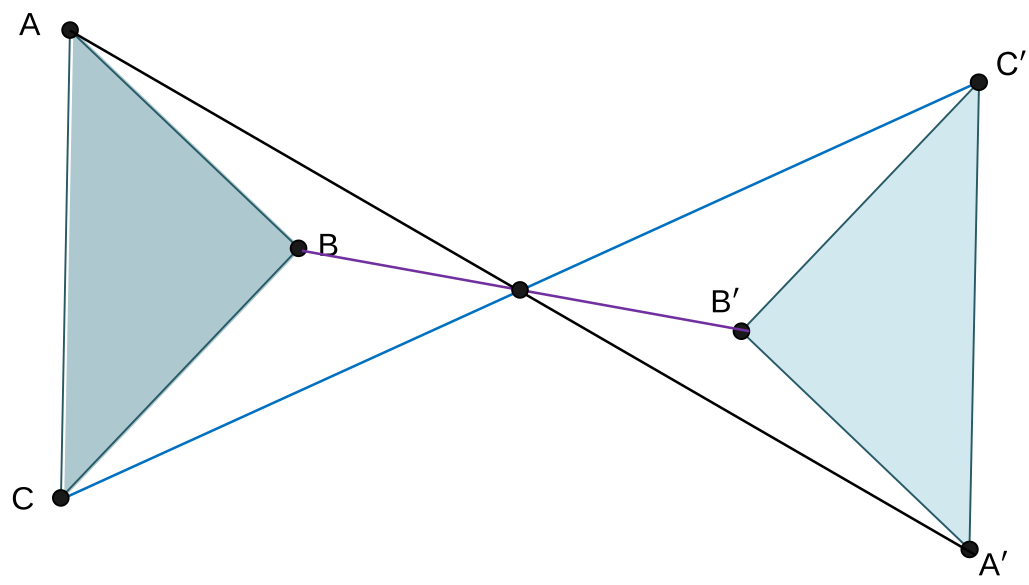
Rotations: Attributes



- The distances connecting corresponding vertices to each other:
  - pass through the center of rotation.
  - are not congruent .
  - are not parallel.
- The distance between each point on the pre-image and the center of rotation is the same as the distance between the corresponding point on the image and the center of rotation.
- All points on the figure are rotated , not just the vertices.

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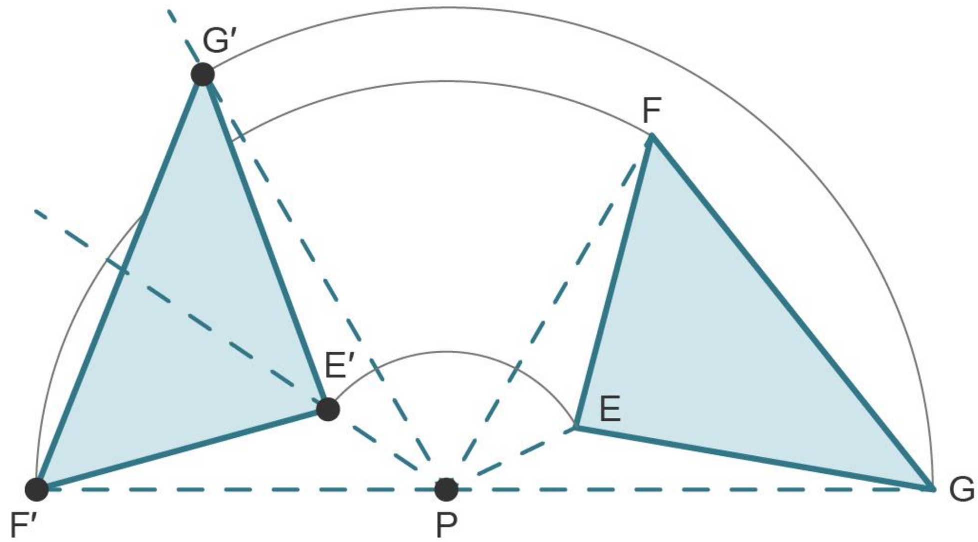
Performing Rotations

Triangle EFG is rotated 120° counterclockwise to form its image, E′F′G′.



To find the image of each vertex:

- Connect the vertex to the center of rotation, P, with a straightedge.
- Move the protractor so that its center is flush with the line drawn and the center of the protractor is aligned with the center of rotation.
- Mark 120° and then draw a dashed guideline to P.
- Place the point of the compass on the center of rotation and the pencil point on the vertex. Draw a curve.
- The intersection of the curve in the guideline is the location of the image of the vertex.

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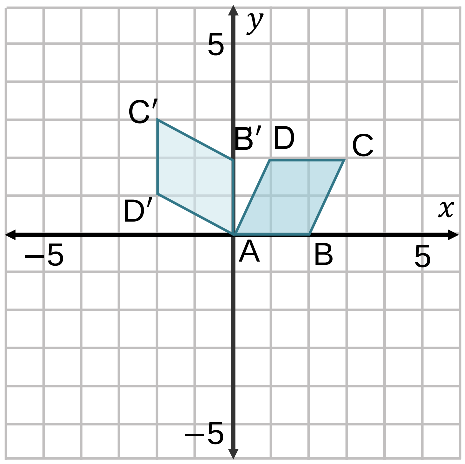
Rotations in the Coordinate Plane

Rotations require:

- Center of rotation
    - The center of rotation is at the vertex **A**.
    - A and A′ have the same position, since the center of rotation doesn't move.
- Angle of rotation
    - The angle created is **90** degrees.



Rotations preserve:

- Angle **measures** of figures
- Side **lengths** of figures

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Rules for Rotating about the Origin

Rotations, $R_{O,\theta}$ in the coordinate plane are considered functions that map, where O, the origin, is the center of rotation and θ is the angle of rotation.

Examples:

- $R_{O,90°}: (x, y) \rightarrow (-y, x)$

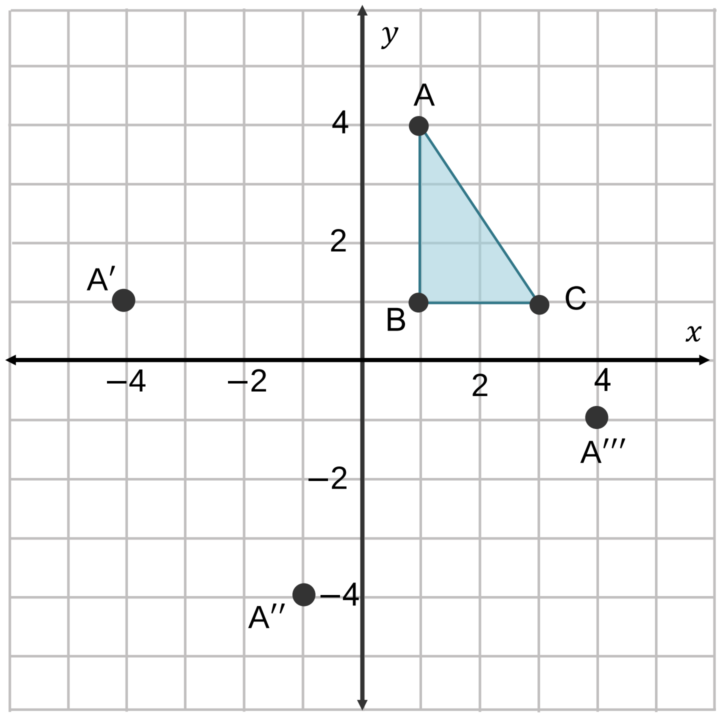
$$A(1, 4) \rightarrow A'(-4, 1)$$

- $R_{O,180°}: (x, y) \rightarrow (-x, -y)$

$$A(1, 4) \rightarrow A''(-1, -4)$$

- $R_{O,270°}: (x, y) \rightarrow (y, -x)$

$$A(1, 4) \rightarrow A'''(4, -1)$$



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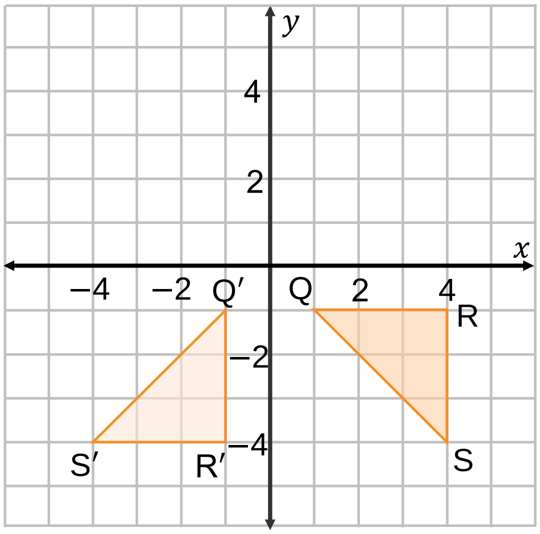
Writing a Rule for a Rotation

Example: What rule describes the rotation about the origin from the pre-image to the image?

$Q(1, -1) \rightarrow Q'(-1, -1)$

$R(4, -1) \rightarrow R'($ −1 $, -4)$

$S(4, -4) \rightarrow S'(-4,$ −4 $)$



Rule:

- $(x, y) \rightarrow (y,$ −x $)$
- $R_{O,\,270}(x, y)$

How many degrees about the origin was the figure rotated?

270 °

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Rotating about the Origin

Example: What is the image of triangle DEF after the rotation $R_{O,\,270°}(x, y)$?

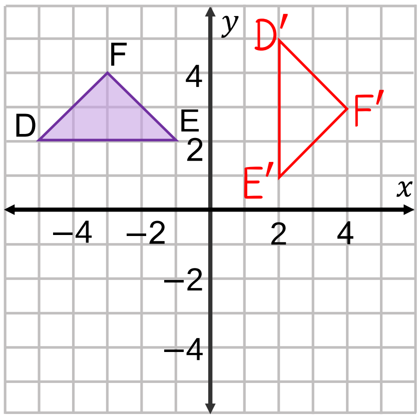
Draw the rotated image and label the vertices

$R_{O,\,270°}: (x, y) \rightarrow ($ y $, -x)$

$D(-5, 2) \rightarrow D'($ 2 $, 5)$

$E(-1, 2) \rightarrow E'(2,$ 1 $)$

$F(-3, 4) \rightarrow F'(4, 3)$



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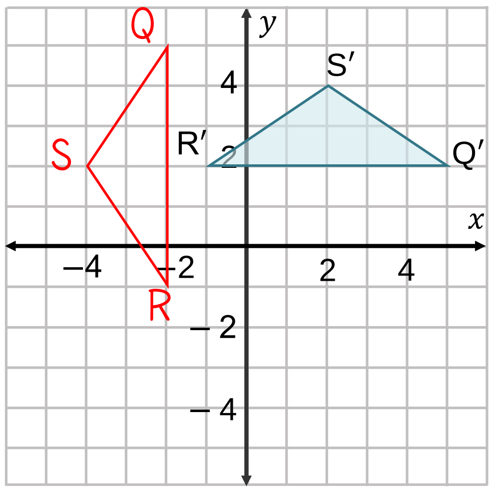
Finding a Pre-Image

Example: What is the pre-image of the figure if the rule that created the image is $R_{0,-90°}(x, y)$?

When given the image, we have to undo the original rotation, so we must go in the opposite direction of the given rule to find the pre-image.

Draw the rotated image and label the vertices



Rule: $(x, y) \to (-y, x)$

$Q(-2, 5) \to Q'(5, 2)$

$R(-2, -1) \to R'(-1, 2)$

$S(-4, 2) \to S'(2, 4)$

Summary | Rotations

Lesson Question

How are rotations defined mathematically?

Answer

(Sample answer) Rotations are defined by rules describing the angle of rotation and the center of that rotation.

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Review: Key Concepts

- Rotations are rigid transformations.
- The center of rotation does not change position.
- The angle of rotation is counterclockwise when positive and clockwise when negative.
- Segments connecting corresponding vertices to the center of rotation are congruent.
- Orientation is not preserved other than for a rotation of 360°.

RULES FOR ROTATIONS AROUND THE ORIGIN

- $R_{O,\,90°}: (x, y) \to (-y, x)$
- $R_{O,\,180°}: (x, y) \to (-x, -y)$
- $R_{O,\,270°}: (x, y) \to (y, -x)$

Summary | Rotations

Use this space to write any questions or thoughts about this lesson.